1 Non-renewable resources and climate change: Are we back to zero growth?

We have seen that the Solow model overcomes the Malthusian logic of poverty traps. Different from the Malthus model, the Solow model suggests that there are no limits to growth: even in its steady state, output per worker grows at a constant exponential rate. We have seen that the data of developed economies over the last century is consistent with this prediction. Nevertheless, over the last decades, many economists have come back to the question whether permanent growth in output per worker is indeed possible. The basic idea always comes back to the basic Malthusian insight: permanent growth is difficult to sustain when some factor of production is finite. In the late 1960s, economists were principally concerned with the finiteness of non-renewable resources used in production, such as oil. We will see that these concerns were likely misguided. Even when a resource that is essential in the production process is finite, constant economic growth is still the likely outcome. Moreover, looking at price data suggests what we think are essential non-renewable resources are either non-essential or are practically in infinite supply. Since the 2000s, the concern of economists has shifter towards the environment as a finite resource. First, we begin by considering pollution that arises as a by-product of the production process. Here, we will see that technological progress against promises hope in allowing for constant growth in output per worker without ever increasing pollution levels. Indeed, the data is consistent with such a model as we see pollution per unit of output declining in the data over time. Finally, we consider the case of green-house gas emissions and discuss why these may pose a novel challenge to constant economic growth in the future.

1.1 Non-renewable resources

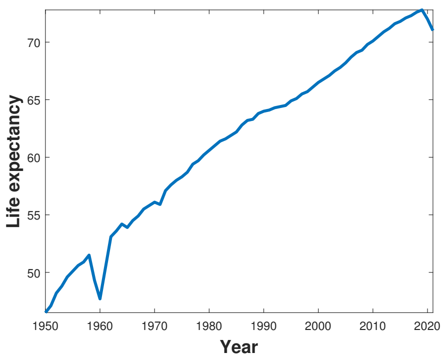
Starting in the late 1960s, economists started to worry that there may be a limit to economic growth given that some factors of production are non-renewable. One particular famous example was Meadows et al. (1972), who, in their contribution for the *Club of Rome*, conducted computer simulations for world output and population. They wrote

"*Given present resources consumption rates and the projected increase in the rates, the great majority of the currently important nonrenewable resources will be extremely costly 100 years from now. [...] The prices of those resources with the shortest static reserve indices have already begun to increase. The price of mercury, for example, has gone up 500 percent in the last 20 years; the price of lead has increased 300 percent in the last 30 years.*"

The argument, certainly, has a lot of intuitive appeal to it. If mercury is essential in the production process of a modern economy, once we have used-up all mercury that exists on earth, we can no longer produce goods and services. In a sense, an economy with a non-renewable resource is even worse for economic growth than the Malthus economy where land was fixed but not used-up over time. Given the prediction of economic growth slowing down, Ehrlich (1968) revived the Malthusian logic of a population growing faster than food supply writing:

"*The battle to feed all of humanity is over. In the 1970s and 1980s hundreds of millions of people will starve to death [...]. At this late date nothing can prevent a substantial increase in the world death rate.*"

Figure 1: Life expectancy over time



Source: United Nations

It is needless to day that these dooms-day predictions did not come to pass. Figure 1 displays the life expectancy in the world over time. Neither during the 1970s no the 1980s did we observe mass starvation around the world. To explain the continuing process of constant economic growth, this chapter develops a model based on the Solow model but with a non-renewable resource that is essential in the production process that, nevertheless, features constant economic growth in its steady state.

1.1.1 Model set up

Assume production is given by

$$Y(t) = A(t)^{1-\alpha} K(t)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma}, \tag{1}$$

where $E(t)$ is the amount of the non-renewable resource used in production. Note, the function has constant returns to scale in $K(t), E(t), L(t)$. As in the Solow model, there are different (but economically equivalent) ways to have $A(t)$ in the production function. Here, it enters with the same exponent, $1 - \alpha$, as in the basic Solow model which will make the comparison across the models simpler. The factors of production that we also have in the Solow model have the same dynamics over time as before:

$$\frac{\dot{L}(t)}{L(t)} = n, \tag{2}$$

$$\frac{\dot{A}(t)}{A(t)} = g, \tag{3}$$

$$\dot{K}(t) = sY(t) - \delta K(t). \tag{4}$$

Next, we need to think about the dynamics of the non-renewable resource, $E(t)$. Assume we start in period 0 with a stock of the non-renewable resource $R(0)$. For example, this could be the barrels of oil that are in the earth. Our use of the resource depletes this stock over time:

$$\dot{R}(t) = -E(t). \tag{5}$$

One can show that when competitive firms own the resource, optimal behavior implies that each period a constant fraction of the remaining stock is used:

$$s_E = \frac{E(t)}{R(t)}, \tag{6}$$

where s_E is the extraction rate. For example, if $s_E = 0.01$ and we start with 100 barrels of oil, than in the first period we will use $E(1) = 1$, and in the second period, we will use $E(2) = 0.01*99 = 0.99$ and so on. Combining equations (5) and (6) shows that the stock will decline over time at rate s_E:

$$\frac{\dot{R}(t)}{R(t)} = -s_E = \frac{\dot{E}(t)}{E(t)}, \tag{7}$$

where the second equality follows from taking logs and derivatives with respect to time of (6). A stock declining at a constant rate s_E implies an exponential growth process for the stock:

$$R(t) = R(0)\exp(-s_E t). \tag{8}$$

Hence, we know also that consumption of the resource is declining exponentially over time:

$$E(t) = s_E R(0)\exp(-s_E t). \tag{9}$$

1.1.2 The steady state of the economy

To analyze the steady state, we follow teh same steps as in the Solow model. That is, we first find an expression for the capital-to-output ratio using the production function:

$$z(t) = \frac{K(t)}{Y(t)} = \frac{K(t)^{1-\alpha}}{A(t)^{1-\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma}} \tag{10}$$

Next, find its growth rate:

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)g + \gamma s_E - (1-\alpha-\gamma)n \tag{11}$$

and assume a steady state where the capital-to-output ratio is constant:

$$0 = \left(\frac{\dot{K}(t)}{K(t)}\right)^* - g + \frac{\gamma}{(1-\alpha)}s_E - \frac{(1-\alpha-\gamma)}{(1-\alpha)}n \tag{12}$$

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^* = g + \frac{(1-\alpha-\gamma)}{(1-\alpha)}n - \frac{\gamma}{(1-\alpha)}s_E \tag{13}$$

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^* = n + g - \frac{\gamma}{(1-\alpha)}(n + s_E) \tag{14}$$

which is our first steady state condition. For the capital-to-output ratio to be constant, the capital stock has to grow at rate $n + g - \frac{\gamma}{(1-\alpha)}(n + s_E)$. Note, this growth rate is lower than in the standard Solow model, $n + g$. The reason is that, in the presence of a non-renewable resource, population growth and resource extraction slow down output growth over time. The reason for the latter is straightforward: A higher extraction rate implies that the resource use declines faster over time, see (9), leading to a slowdown in output growth. The negative effect of population growth is more subtle. As in Malthus, a growing population decreases labor productivity given that at least one factor of production cannot adjust.

We obtain our second steady state condition from the capital accumulation equation:

$$\dot{K}(t) = sA(t)^{1-\alpha}K(t)^{\alpha}E(t)^{\gamma}L(t)^{1-\alpha-\gamma} - \delta K(t) \tag{15}$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{s}{z(t)} - \delta, \tag{16}$$

which is the same as in the Solow model. Putting things together,

$$n + g - \frac{\gamma}{(1-\alpha)}(n + s_E) = \frac{s}{z^*} - \delta \tag{17}$$

$$z^* = \frac{s}{n + g + \delta - \frac{\gamma}{(1-\alpha)}(n + s_E)}, \tag{18}$$

which is indeed constant, i.e., depends only on model parameters. Note, for the same s, n, g, δ, the steady state capital-to-output ratio is higher than in the Solow model. The reason is, discussed above, in the presence of a non-renewable resource, population growth and resource extraction will slow down output growth over time.

Once we have the steady state capital-to-output ratio, we can obtain output in steady state by rewriting output as a function of the capital-to-output ratio:

$$Y(t) = A(t)^{1-\alpha} K(t)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma} \tag{19}$$

$$Y(t)^{1-\alpha} = A(t)^{1-\alpha} \left(\frac{K(t)}{Y(t)}\right)^{\alpha} E(t)^{\gamma} L(t)^{1-\alpha-\gamma} \tag{20}$$

$$Y(t) = A(t) \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} E(t)^{\frac{\gamma}{1-\alpha}} L(t)^{1-\frac{\gamma}{1-\alpha}} \tag{21}$$

$$Y(t) = A(t) \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \left(s_E R(0) \exp(-s_E t)\right)^{\frac{\gamma}{1-\alpha}} L(t)^{1-\frac{\gamma}{1-\alpha}} \tag{22}$$

Finally, dividing both sides by the number of workers yields output per worker in steady state:

$$y(t)^* = \left(\frac{s}{n+g+\delta-\frac{\gamma}{(1-\alpha)}(n+s_E)}\right)^{\frac{\alpha}{1-\alpha}} \left(s_E R(0) \exp(-s_E t)\right)^{\frac{\gamma}{1-\alpha}} L(t)^{-\frac{\gamma}{1-\alpha}} A(t) \tag{23}$$

Note, the depletion rate s_E enters three times into the expression. A higher depletion rate (i) increases the capital-to-output ratio, (ii) raises the resource use and, thereby, production, and (iii) reduces the stock of resources over time and, thereby the resource use.

1.1.3 Growth in steady state

The key question is whether the economy can deliver constant long-run economic growth despite the non-renewable resource. Taking logs and the derivative with respect to time of (22) yields

$$\ln Y(t)^* = \ln A(t) + \frac{\alpha}{1-\alpha} \ln \left(\frac{K(t)}{Y(t)}\right)^* + \frac{\gamma}{1-\alpha} \left(\ln(s_E R(0)) - s_E t\right) + \left(1 - \frac{\gamma}{1-\alpha}\right) \ln L(t)$$

$$\left(\frac{\dot{Y}(t)}{Y(t)}\right)^* = g + n - \frac{\gamma}{1-\alpha}(s_E + n).$$

As in the Solow model, output grows with $n+g$. However, as discussed above, the non-renewable resource creates a drag on output growth over time, $-\frac{\gamma}{1-\alpha}(s_E+n)$. Turning to the growth rate of output per worker, we have Instead of total output, we can also look at output per capita:

$$\left(\frac{\dot{y}(t)}{y(t)}\right)^* = g - \frac{\gamma}{1-\alpha}(s_E+n).$$

Note, the depletion rate has the same negative effect on the growth rate of output per worker as the population growth rate. Both reduce the efficiency of labor over time. We have positive growth in GDP per worker iff

$$g > \frac{\gamma}{1-\alpha}(s_E+n).$$

That is, with a positive growth of technology, we can still have constant growth of output per worker in steady state. The reason is simple: Though the amount of the non-renewable resource used in production is declining, the other two factors of production are still growing at rates

$$\frac{\dot{A}(t)}{A(t)} = g, \tag{24}$$

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^* = n + g - \frac{\gamma}{(1-\alpha)}(n+s_E). \tag{25}$$

This growth is able to overcome the negative growth in $E(t)$ leading to constant growth in output per worker. Using the example of mercury, we have less an less mercury over time to put into light bulbs but we also develop new designs of light bulbs that use less and less mercury per light bulb allowing us to increase the total production of light bulbs over time. This simple example is one way to think about the fact that we observe continuing constant output per worker growth over time, though, as the next section shows, probably not the best way. A better way to think about it is that over time, we simply stop using mercury for light bulbs and switch to LED lights, i.e., mercury is simply not an essential factor of production after all.

1.1.4 Price growth in steady state

One way to test the present model is to look at the behavior of the price of the non-renewable resource over time. To derive this price behavior, note that, given our Cobb-Douglas production function, the share of income going to non-renewables should be constant over time:

$$
\begin{aligned}
P_E(t)E(t) &= \gamma Y(t) \\
P_E(t) &= \gamma \frac{Y(t)}{E(t)}.
\end{aligned}
$$

Now take logs and the derivative with respect to time to obtain the growth rate of the non-renewable price:

$$
\begin{aligned}
\frac{\dot{P}_E(t)}{P_E(t)} &= \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{E}(t)}{E(t)} \\
\frac{\dot{P}_E(t)}{P_E(t)} &= g - \frac{\gamma}{1-\alpha} s_E + \left(1 - \frac{\gamma}{1-\alpha}\right) n + s_E \\
\frac{\dot{P}_E(t)}{P_E(t)} &= g + \left(1 - \frac{\gamma}{1-\alpha}\right)(n + s_E) \\
\frac{\dot{P}_E(t)}{P_E(t)} &= g + \frac{1-\alpha-\gamma}{1-\alpha}(n + s_E) > 0
\end{aligned}
$$

The price of non-renewables rises over time for three reasons. First, technological progress raises the marginal product of non-renewables over time. Second, population growth raises the marginal product of non-renewables over time. Third, the falling stock of non-renewables raises its marginal product over time. Because our model is a real model, instead of studying the growth in the price of non-renewables over time, it is easier to study it relative to another price. Here, we use the price of labor. Given constant factor shares, we have:

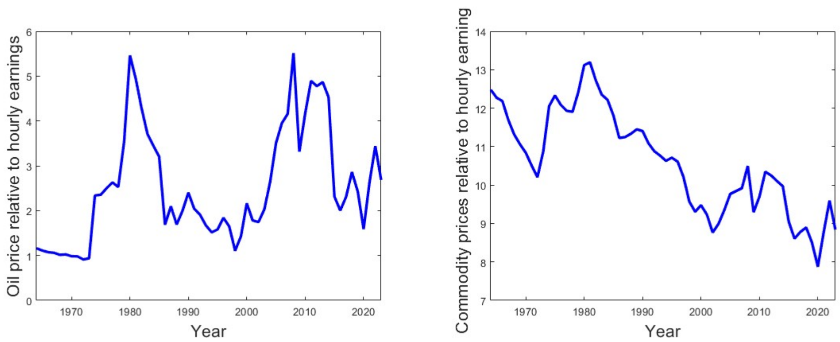
$$
\begin{aligned}
\frac{P_E(t)E(t)}{w(t)L(t)} &= \frac{\gamma Y(t)}{(1-\gamma-\alpha)Y(t)} \\
\frac{P_E(t)}{w(t)} &= \frac{\gamma}{(1-\gamma-\alpha)} \frac{L(t)}{E(t)} = \frac{\gamma}{(1-\gamma-\alpha)} \frac{L(0)\exp(nt)}{s_E R(0)\exp(-s_E t)}
\end{aligned}
$$

Next, take logs and the derivative with respect to time to get the growth rate in the price wage ratio, $RP(t) = \frac{P_E(t)}{w(t)}$:

$$\frac{\dot{R}P(t)}{RP(t)} = n + s_E.$$

With $n > 0$, resources become more scarce over time relative to labor implying that their relative price is growing. Figure 2 displays the ratio in the data. The left panel displays the price of oil relative to wages in the U.S. over time. The right panel uses a broader measure of a commodity basket. The figure shows that, instead of rising prices for non-renewables relative to wages, we have, if any, falling prices.

Figure 2: Relative price of non-renewables



Source: St. Louis Fed

This contradiction raises the question what model assumption is incorrect. The model makes two key assumptions. First, the resource is depleting over time. In a theoretical sense, this must be true: the amount of barrels of oil in the earth is, indeed, finite. However, economically speaking, the theoretical amount of the resource is of little importance. What matters is how much we can extract given current conditions. As extraction technology improves, think of technological improvements that made shale gas extraction economically profitable, the amount of available resources may increase over time instead of decrease. Figure 3, taken from Blackman and Baumol (2008), provides some numbers supporting this idea. Despite using all the listed resources over the last decades, today's reserves of those resources are larger than they were in the 1950s.

Figure 3: Resource availability

Mineral	1950 Reserves	Production 1950–2000	2000 Reserves
Tin	6	11	10
Copper	100	339	340
Iron Ore	19,000	37,583	140,000
Lead	40	150	64
Zinc	70	266	190

Source: Blackman and Baumol (2008)

The second key assumption that the model made is that the resource is essential in the production process. Again, the assumption ignores how technological progress, together with market forces, can change things over time, in this case the production process. Simon (1980) was an early critique of theories relying on essential, non-renewable resources. He provides a good example from history: In the 16^{th} century, most ships were build out of wood leading to deforestation of large parts in Europe. As a result, the price of wood rose leading to incentives to innovate by using other materials. Over time, ships were build out of iron and later steel. What is more, we invented ways to recycle these resources. The above discussion about using LED lamps to avoid the use of mercury is another example in this vein.

1.2 A green Solow Model

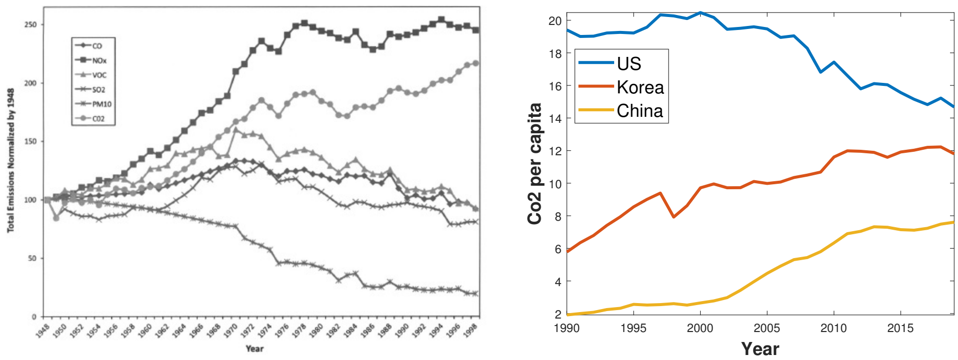
Over the last decades, economists started to worry about permanent economic growth in light of the environmental damage that is often associated with production. If one thinks about the environment as a non-renewable resource, pollution depletes it over time. Different from other resources, it is most natural to think of the environment as a consumption good, i.e., it does not directly enter the production process. Importantly, we can pay resources to avoid pollution. For example, we may use production technologies that require less pollution but are more expensive, such as more efficient internal combustion engines in a car. Moreover, we can switch to different energy sources that produce less pollution but are more expensive, such as moving from coal to wind energy production. Finally, we may switch away from goods that create a lot of pollution, such as air travel, to a good that pollutes less, such as bus travel. This chapter follows the paper from Brock and Taylor (2010) who present data on pollution and then a model that allows us to rationalize the data. The key takeaway will be that, because of technological progress, constant long-run economic growth in the will still be feasible without ever rising pollution levels.

1.2.1 Data about pollution

Brock and Taylor (2010) highlight three facts about pollution over time that we will go through. First, pollution increases initially with per capita income but starts falling at some point. As this behavior resembles the one of cross-sectional inequality within a country first described by Kuznets, this behavior is coined an environmental Kuznets curve. The left panel of figure 4 displays the behavior for the U.S. For a rich country like the U.S., despite substantial output per worker growth over time, many pollutant emissions are falling since 1984. The point when a pollutant reaches its maximum may differ across pollutants. While CO2 continued to rise until the late 90s, VOCs (Volatile Organic Compounds) started falling at the beginning of the 70s. The right panel looks at one particular emission, CO2 emissions, for three countries at different levels of economic development. In the U.S., since the late 1990s, total emissions relative to the population are declining. Note, pollution per capita are highest in the U.S. suggesting a positive relationship between output and pollution. Different from the U.S., in countries with less development like South Korea and China, emissions are still growing until today though they are close to flat in South Korea since 2010 suggesting that South Korea might have reached already its peak.

The fact that pollution levels are declining in rich countries despite economic growth may be surprising. However, it simply reflects the fact that societies care about the environment and are willing to pay resources to avoid pollution, something called abatement. A good example comes from the U.S. during the 1950s and 1960s. In the state of Ohio, one of the most industrial states, a major transportation river, the Cuhayoga, was so polluted with chemicals that the river was

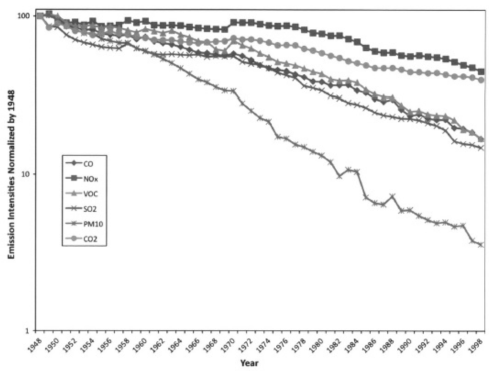
Figure 4: Environmental Kuznets curve



Source: Brock and Taylor (2010) and World Bank

burning on several occasions. This created such outrage, that it was a major contributor for the federal government to found the employment protection agency whose role is to monitor air and water pollution and which has regular authority to impose production standards on regulated industries. The result was a marked improvement in water and air quality with the Cuhayoga being much less polluted today than during the 1960s.

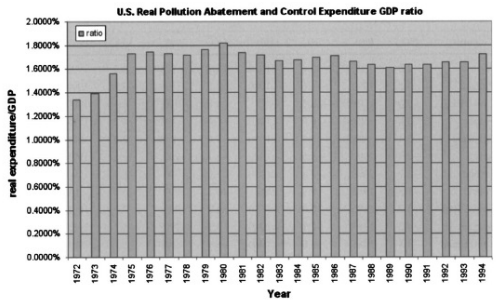
Figure 5: Pollution intensity



Source: Brock and Taylor (2010)

The second fact that the they highlight is that irrespective of the pollutant, the pollution intensity, that is the pollution per unit of output produced is falling over time, as Figure 5 shows. Note, the pollution intensity is falling at a close to constant rate over time. A falling pollution intensity over time may suggest that the environment is a luxury good, i.e., as we become richer, we are willing to give up more resources to protect the environment. Figure 6 shows that this is inconsistent with the data on abatement costs. Despite rising income, the share of income spend on abatement is close to constant, around 1.7%, over time, which is the third data fact from Brock and Taylor (2010). If the environment was a luxury good, we would expect its expenditure share to increase as income grows. How is it possible that the pollution intensity is falling despite the expenditure share of abatement being constant? A natural interpretation is that there is technological progress leading to less pollution per unit of production over time. Moreover, the data suggests that the improvement occurs at a constant rate.

Figure 6: Abatement costs



Source: Brock and Taylor (2010)

1.2.2 Model setup

The model we use to rationalize these facts is based on the Solow model without education. We add to this model that production produces pollution and that we can pay resources to reduce this pollution. Moreover, as seen in the data, the amount of pollution per unit of production is decreasing over time. To be specific, output is produced according to

$$Y(t) = K(t)^{\alpha} \left(A(t)L(t)\right)^{1-\alpha}, \tag{26}$$

with our familiar laws of motions:

$$\frac{\dot{L}(t)}{L(t)} = n \tag{27}$$

$$\frac{\dot{A}(t)}{A(t)} = g. \tag{28}$$

Turning to the new part of the model, the pollution, we assume that each unit of output Y creates Ω units of pollution. However, part of the pollution can be reduced by abatement such that emitted pollution is:

$$E(t) = Y(t)\Omega(t) - \Omega(t)B(t), \tag{29}$$

where $B(t)$ is the abatement technology. The amount of abatement depends on the amount of pollution $\Omega(t)Y(t)$ and the effort we put into abatement, $\theta Y(t)$. As shown above, a constant fraction of output that is spent on abatement is a good approximation. We assume that we can write the abatement function as a constant returns to scale function depending on total output and the effort we put into abatement, $\theta Y(t)$:

$$B(t) = B(Y(t), \theta Y(t)). \tag{30}$$

Hence, the amount of emissions is given by

$$E(t) = Y(t)\Omega(t) - \Omega(t)B(Y(t), \theta Y(t)). \tag{31}$$

As B has constant returns to scale, we can write emissions and the emission intensity, respectively as:

$$E(t) = Y(t)\Omega(t)\left[1 - B(1,\theta)\right] \tag{32}$$

$$\frac{E(t)}{Y(t)} = \Omega(t)\left[1 - B(1,\theta)\right]. \tag{33}$$

Recall, in the data, $\frac{E(t)}{Y(t)}$ is decreasing at a constant rate, and θ is constant. Hence, to match the data, we need $\Omega(t)$ to grow at a constant negative rate:

$$\Omega(t) = \Omega(0)\exp(-g_B t). \tag{34}$$

Put differently, the amount of emissions resulting from a unit of output is decreasing over time. You may think of this as technological progress, e.g., solar energy becoming cheaper and, hence, the economy switches to more solar energy production over time.

The only equationwe are still missing are the dynamics of capital accumulation. This equation is almost the same as in the Solow model but we have to take into account that we use part of production, θ, for abatement. Hence, the amount of output left for consumption and investment is:

$$I(t) + C(t) = (1-\theta)Y(t). \tag{35}$$

As a result, we obtain a slightly modified law of motion for capital:

$$\dot{K}(t) = (1-\theta)sK(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} - \delta K(t). \tag{36}$$

1.2.3 The steady state

To find a steady state, we proceed as always. First, we derive the capital-to-output ratio from the production function:

$$z(t) = \frac{K(t)}{Y(t)} = \frac{K(t)}{K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha}} \tag{37}$$

$$= \left(\frac{K(t)}{A(t)L(t)}\right)^{1-\alpha}. \tag{38}$$

Next, we find the growth rate of the capital stock in steady state assuming that the capital-to-output ratio is constant:

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)(n+g) \tag{39}$$

$$\left(\frac{\dot{K}(t)}{K(t)}\right)^* = n+g \tag{40}$$

Next, we obtain the second steady state condition from the capital accumulation equation:

$$\dot{K}(t) = s(1-\theta)K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} - \delta K(t) \tag{41}$$

$$\frac{\dot{K}(t)}{K(t)} = \frac{s(1-\theta)}{z(t)} - \delta, \tag{42}$$

and finally we put the two together to solve for the steady state capital-to-output ratio:

$$z^* = \left(\frac{K(t)}{Y(t)}\right)^* = \frac{s(1-\theta)}{n+g+\delta}. \tag{43}$$

Now we can use the production function and the fact that consumption per worker is $c(t) = (1-s)(1-\theta)y(t)$ to solve for output and consumption per worker in the steady state:

$$y(t)^* = \left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t) \tag{44}$$

$$c(t)^* = (1-s)(1-\theta)\left(\frac{s(1-\theta)}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} A(t). \tag{45}$$

The equations highlight two economic costs of abatement effort. First, a higher abatement effort reduces consumption per worker because abatement itself is costly leaving less output that is available for consumption. Second, for the same reason, also investment is lower and, hence, the steady state capital-to-output ratio is lower which reduces the steady state output per worker and, hence, steady state consumption per worker.

We are now ready to think about pollution growth in steady state. As $E(t) = Y(t)\Omega(t)\left[1 - B(1,\theta)\right]$, the growth rate of emissions, g_E, is determined by the sum of the growth rate of output and the growth rate of emissions created by output, $\Omega(t)$. By assumption the latter is equal to a constant $-g_B$. Hence, we only need the steady state growth rate of output, which is the same as in the Solow model:

$$Y(t) = \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} A(t)L(t) \tag{46}$$

$$\Rightarrow \left(\frac{\dot{Y}(t)}{Y(t)}\right)^{*} = n + g. \tag{47}$$

Hence, we have in steady state

$$g_E^{*} = \left(\frac{\dot{E}(t)}{E(t)}\right)^{*} = n + g - g_B. \tag{48}$$

Whether total emissions fall in steady state depends on the race between output growth and the growth rate of emissions per output. As we have seen, the U.S. data suggests that in steady state, total emissions fall, i.e., $g_B > n + g$.

1.2.4 Transition dynamics and the Kuznets curve

The steady state result can explain why developed economies have falling emission levels. The model explains the rising emission levels at lower levels of development through transition dynamics. To derive the growth rate of emissions outside the steady state, we need to find the growth rate of output outside the steady state

$$Y(t) = \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} A(t)L(t) \tag{49}$$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)} + g + n. \tag{50}$$

Substituting this into the growth rate of emissions, we can write this growth rate as

$$\frac{\dot{E}(t)}{E(t)} = g_E^* + \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)}. \tag{51}$$

The intuition is simple: When the capital-to-output ratio grows, output grows and, thus, emissions grow faster than in steady state. Hence, countries that are far below their steady state will experience a rapid rise in the capital-to-labor ratio and, thus, a rapid output growth and a high growth rate of emissions. As countries converge to their steady state, output growth slows down and so does emissions growth.

Figure 7: Pollution dynamics

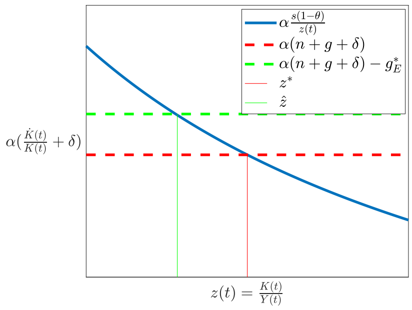


Figure 7 shows this idea graphically. At the steady state capital-to-output ratio z^*, we have that $\alpha\frac{s(1-\theta)}{z(t)} = \alpha(\delta+n+g)$, and $\frac{\dot{E}(t)}{E(t)} = g_E^*$. To the right of z^*, the growth rate of output is below $n+g$ and, hence, $\frac{\dot{E}(t)}{E(t)} < g_E^*$. To the left of z^*, output growth and, hence, emission growth increase. There exists some $\hat{z}$ where emission growth is zero, i.e., when the additional output growth coming from a rising capital-to-output growth is equal to the steady state (negative) growth rate in emissions: $\frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)} = -g_E^*$ To compute that point, we rewrite the dynamics of the capital-to-output ratio:

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{\dot{K}(t)}{K(t)} - (1-\alpha)(n+g) \tag{52}$$

$$\frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)} = \alpha\frac{\dot{K}(t)}{K(t)} - \alpha(n+g), \tag{53}$$

and substitute $\alpha \frac{\dot{K}(t)}{K(t)} = \alpha \left(\frac{s(1-\theta)}{z(t)} - \delta \right)$ to obtain

$$\frac{\alpha}{1-\alpha} \frac{\dot{z}(t)}{z(t)} = \alpha \frac{s(1-\theta)}{z(t)} - \alpha \left(n + g + \delta \right). \tag{54}$$

Hence, at $\hat{z}$, $\alpha \frac{s(1-\theta)}{z(t)} - \alpha \left(n + g + \delta \right) = -g_E^*$. To any point left of $\hat{z}$, output growth is so fast that emissions growth becomes positive, e.g., the cases of South Korea and China.

As in the standard Solow model, we can find again a solution for the growth rate of the capital-to-output ratio. Following the exact same steps, we obtain

$$z(t) = \frac{s(1-\theta)}{n+g+\delta} + \left[z(0) - \frac{s(1-\theta)}{n+g+\delta} \right] \exp(-\beta t) \tag{55}$$

$$\beta = (1-\alpha)(n+g+\delta). \tag{56}$$

Given this solution, consider the transition dynamic for a country that is currently in its steady state and increases its abatement effort. The steady state capital-to-output ratio, $z^* = \frac{s(1-\theta)}{n+g+\delta}$, will fall such that $z(0) > z^*$. As a result, the capital-to-output ratio will fall over time leading to falling output and a temporarily more negative growth rate in emissions. We know that the growth rate of the capital-to-output ratio is particularly negative early in the transition leading to the most negative growth rate in emissions right after the reform and the growth rate of emissions converging monotonically back to $-g_E^*$.

1.3 CO2 emissions and economic damage

Thus far, we have studied the link from production to pollution. Since the 2000s, economists have become particularly concerned with CO2 pollution. One interesting aspect of CO2 pollution is that rising pollution levels may affect negatively output levels. The idea is that CO2 emissions raise global temperatures leading to more droughts and, thus, less production, more floods and, thus, more economic damages, and more heat-waves and, thus, more deaths. Some people object to this arguing that for each drought in Africa, farm land in Siberia becomes available, for each flooding, the opening passage through the Arctic will reduce shipping costs, and each person of dying is outweigh by fewer people dying of freezing. Economists try to net all those things out to compute the true costs of climate change but there are large uncertainty bounds around, both, by how much CO2 emissions raise temperatures and around how much rising temperatures result in economic damages. What is clear is that the costs of climate change will depend a lot on how good we are in reallocating production to regions that benefit. We will not enter into the discussion on the true economic costs of climate change here but rather take it as given that CO2 emissions cause economic damage and highlight some qualitative trade-offs that arise from emitting CO2.

1.3.1 A model with pollution damages

We assume again that output is produced according to a Cobb-Douglas production function. Importantly, emissions, $E(t)$, increase output:

$$Y(t) = \frac{E(t)^{\gamma}}{\exp(\theta D(t))} K(t)^{\alpha} \left(A(t)L(t)\right)^{1-\alpha-\gamma}. \tag{57}$$

You can think of two ways that emissions increase output. First, emission intensive production processes, such as fossil energy sources, are cheaper and easier to manage than renewables. Second, and in line with the last section, you can think of emissions allowing us to save on abatement costs. Important, emissions now create economic damage, $D(t)$ which reduces productivity by $\frac{1}{\exp(\theta D(t))}$. The reduction of productivity is the net effect that CO2 emissions have on economic output discussed above. New emissions increase the environmental damages, while damages depreciate at a constant rate δ_D:

$$\dot{D}(t) = E(t) - \delta_D D(t). \tag{58}$$

You can think of δ_D as the natural depreciation of emissions in the air through plant absorption. However, δ_D may be also man made through technologies like carbon capturing. Consistent with the U.S. data on emissions, I will assume emissions are declining over time at a constant rate:

$$\frac{\dot{E}(t)}{E(t)} = -g_E. \tag{59}$$

Finally, the laws of motion for the population, technology, and capital are:

$$\frac{\dot{L}(t)}{L(t)} = n, \tag{60}$$

$$\frac{\dot{A}(t)}{A(t)} = g, \tag{61}$$

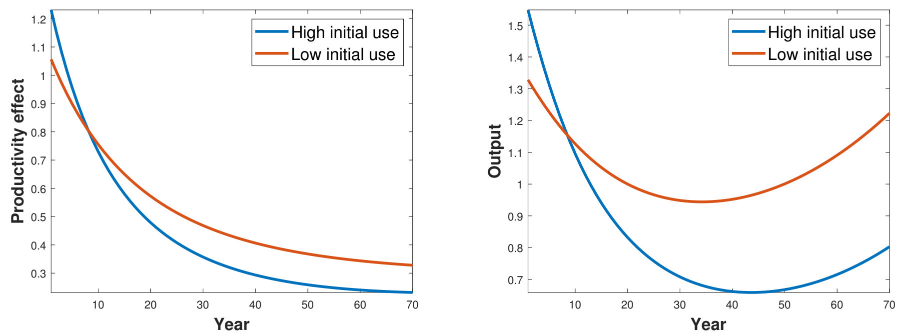
$$\dot{K}(t) = sY(t) - \delta K(t). \tag{62}$$

1.3.2 Trade-offs in the model

Analyzing the steady state of the model, given that we assume emissions are declining over time, is not particularly interesting. With $\frac{\dot{E}(t)}{E(t)} = -g_E$, we have $E(t) \mapsto 0$ and, hence, $D(t) \mapsto 0$, and the model is the same as the one with a non-renewable resource which use rate declines at a constant rate. Instead, the economic interesting aspect of the model is the behavior as we converge to this steady state. Unfortunately, solving this convergence path explicitly is difficult, and we will rely instead on numerical simulations. Note, the effect of emissions on output over time is given by

$$\frac{E(t)^{\gamma}}{\exp(\theta D(t))}. \tag{63}$$

Figure 8: The effect of emissions over time



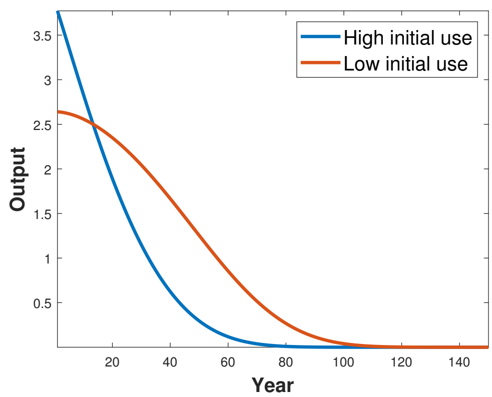
The left panel of Figure 8 displays this productivity effect. A high initial emission level increases productivity during early years, however, as damage accumulates faster, at some point, productivity is higher in the economy that starts with the lower emission level. The right panel displays the resulting output of the two economies. The economy with the initial high level of emissions has a higher output initially but a lower output in the long run. This trade-off between output today vs. the future directly leads to the political discussion on whether, and by how much, we should reduce emissions today. The above discussion makes clear that the answer depends on the question how much we value resources (consumption) today relative to the future. The most prominent economic climate change models suggest that we have to have low discount rates to justify the costs of emission reduction. To see why, note that the costs of emissions today really show up in 50 to 100 years time. With a standard time discount rate of 4%, we have $0.96^{100} < 0.02$, i.e., a consumption reduction in 100 years is valued at 2% of a consumption reduction today. Ultimately, the question of how we should discount the future is a political question that economists don't have particular expertise in. One can make a good moral argument that we ought to value future generations just as much as current generation, i,e., do not apply any discount and maximize well-being in the long run. However, economists can contribute to the debate and point out that with these social preferences, the current generation also ought to increase the savings rate to bring the economy in line with the Golden rule which also relies on a zero time discounting assumption.

1.3.3 Are we back to zero growth?

The problem of overusing a factor sounds very familiar to models of fixed (finite) factors. Yet, we have overcome the Malthus poverty trap and the scarcity of other finite factors of production. Should we expect the same with pollution? In the abstract, we can again overcome the scarcity problem by using green energies or abatement. Just as in the Solow model, this is something we can invest in (no longer a fixed factor) and theoretically in infinite supply (the sun). What makes the problem more difficult are missing property rights. With other non-renewable resources, prices rise when the resource experiences shortage. With pollution, we

have a *tragedy of the common*. It is fair to say that the tragedy of the common also applies to water and air pollution that we studied above. What makes the problem more difficult is that those types of pollutions could be solved at a national level, e.g., by founding the federal EPA. The problem of CO2 emissions is a global problem with no global government that could impose laws and regulations. In fact, over the last decades, we have seen that any attempts to reach an international consensus on CO2 emissions has been elusive. One may speculate that, without such a consensus, emission growth may be positive, instead, as assumed above, negative. Figure 9 displays the effects on output over time when emission growth is growing. In the choosen simulation, the damage function grows fast enough such that output is falling over time. Reflecting these worries, the Nobel price winning economist Nordhaus warns in Nordhaus et al. (1992) to translate lessons from other non-renewables one-to-one to the case of green-house gases:
"*Economists have often belied their tradition as the dismal science by downplaying both earlier concerns about the limitations from exhaustible resources and the current alarm about potential environmental catastrophe. However, to dismiss today's ecological concerns out of hand would be reckless. Because boys have mistakenly cried wolf in the past does not mean that the woods are safe.*"

Figure 9: Output with positive emission growth



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